Origin of holographic dark energy models

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Abstract

We investigate the origin of holographic dark energy models which were recently proposed to explain the dark energy-dominated universe. For this purpose, we introduce the spacetime foam uncertainty of $\delta l \geq l_{\rm p}^{\alpha} l^{\alpha-1}$. It was argued that the case of $\alpha = 2/3$ could describe the dark energy with infinite statistics, while the case of $\alpha = 1/2$ can describe the ordinary matter with Bose-Fermi statistics. However, two cases may lead to the holographic energy density if the latter recovers from the geometric mean of UV and IR scales. Hence the dark energy with infinite statistics based on the entropy bound is not an ingredient for deriving the holographic dark energy model. Furthermore, it is shown that the agegraphic dark energy models are the holographic dark energy model with different IR length scales.

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1 Introduction

Observations of supernova type Ia suggest that our universe is accelerating [1]. Considering the ΛCDM model [2, 3], the dark energy and cold dark matter contribute $\Omega_\Lambda^{\rm ob} \simeq 0.74$ and $\Omega_{\rm CDM}^{\rm ob} \simeq 0.22$ to the critical density of the present universe. Recently, the combination of WMAP3 and Supernova Legacy Survey data shows a significant constraint on the equation of state (EOS) for the dark energy, $w_{\rm ob} = -0.97^{+0.07}_{-0.09}$ in a flat universe [4, 5].

Although there exist a number of dark energy models [6], the two promising candidates are the cosmological constant and the quintessence scenario [7]. The EOS for the latter is determined dynamically by the scalar or tachyon.

On the other hand, there exist interesting models of the dynamical dark energy which satisfy the holographic principle but have different origins. One is the holographic dark energy model [8, 9] and the other is the agegraphic dark energy model [10]. The first is derived from the energy bound [11, 12], while the latter is based on the Károlyházy relation of quantum fluctuations of time [13, 14, 15] and the time-energy uncertainty [16]. It seems that the agegraphic dark energy density is clearly understood because its energy is just the minimum energy of spacetime fluctuations derived from the time-energy uncertainty. However, the origin of holographic energy density remains unclear and obscure because it was obtained from the energy bound using the black hole.

Recently, Ng[17] has proposed that the entropy bound is designed for deriving the holographic dark energy. On the other hand, the energy bound was used for describing the holographic dark energy [11, 8]. Hence, it is necessary to reexamine the holographic dark energy model based on the energy bound.

In this Letter, we address this issue and explore the connection between holographic and agegraphic dark energy models.

2 Spacetime foam uncertainty

We start with reviewing holographic dark energy model. This model comes from the energy bound [11, 12]

$$E_\Lambda \le E_{BH} \to l^3 \rho_\Lambda \le m_{\rm p}^2 l, \tag{1}$$

where the vacuum energy density is given by $\rho_\Lambda = \Lambda^4$ with the UV cutoff Λ and l is the length scale (IR cutoff) of the system. Choosing the saturation of this bound leads to holographic energy density

$$\rho_\Lambda \sim \frac{m_{\rm p}^2}{l^2} \sim \frac{1}{(l_{\rm p} l)^2} \tag{2}$$

Table 1: Summary of spacetime foam (STF) model [17]. Here HM (RWM) denote holographic (random-walk) models, and B/F represent Bose-Einstein/Fermi-Dirac statistics. A is the area of system.

STF model	distance fluctuations	entropy bound	energy/matter	statistics
HM	$\delta l \geq (l_{\rm p}^2 l)^{1/3}$	A	dark energy	infinite
RWM	$\delta l \geq (l_{\rm p} l)^{1/2}$	$A^{3/4}$	ordinary matter	B/F

We note that the energy bound Eq.(1) implies another entropy bound,

$$S_\Lambda \leq \Big(m_{\rm p}^2 A\Big)^{3/4} \tag{3}$$

with $A = 4\pi l^2$ is the area of system. This is not a covariant entropy bound.

On the other hand, the agegraphic dark energy model is based on the Károlyházy relation of quantum fluctuations of time [13, 14, 15]

$$\delta t = \lambda t_{\rm p}^{2/3} t^{1/3} \tag{4}$$

and the time-energy uncertainty

$$\Delta E \sim t^{-1} \tag{5}$$

in the Minkowiski spacetime. This gives us the agegraphic energy density [16]

$$\rho_{\rm T} \sim \frac{\Delta E}{(\delta t)^3} \sim \frac{m_{\rm p}^2}{t^2}. \tag{6}$$

Furthermore, Ng has proposed the spacetime foam model where the covariant entropy bound,

$$S_\Lambda = \Lambda^3 l^3 \leq S_{BH} = m_{\rm p}^2 l^2 \sim A \tag{7}$$

plays a crucial role for conjecturing the presence of the holographic energy density in Eq.(2). A basic feature of this model comes from the spacetime foam uncertainty [17, 18, 19]

$$\delta l \geq l_{\rm p}^{\alpha} l^{\alpha-1}. \tag{8}$$

Explicitly, the $\alpha = 2/3$ case of holographic uncertainty could describe the dark energy with infinite statistics, while the $\alpha = 1/2$ case of random-walk uncertainty can describe the ordinary matter with Bose-Fermi statistics. The important properties are summarized in Table I.

We are in a position to point out the connection between holographic dark energy and spacetime foam models. Assuming the relation between the UV cutoff and distance uncertainty

$$\Lambda \sim \frac{1}{\delta l}, \tag{9}$$

we derive the holographic uncertainty from the entropy bound Eq.(7) [20] as

$$S_\Lambda \le S_{BH} \to \delta l \ge (l_{\rm p}^2 l)^{1/3} \tag{10}$$

and the random-walk uncertainty from the energy bound Eq.(1) as

$$E_\Lambda \le E_{BH} \to \delta l \ge (l_{\rm p} l)^{1/2}. \tag{11}$$

The above is reasonable because the UV cutoff usually determines the minimal detectable length [21]. Hence it seems that the entropy bound (energy bound) are closely related to the HM (RWM), respectively.

Now we wish to obtain the holographic energy density ρ_Λ from the entropy bound of Eq.(10) and the energy uncertainty. For this purpose, we introduce the delocalized states which have typical Heisenberg energy [1]

$$E^{\rm B}_{\rm del} \sim \frac{1}{l} \tag{12}$$

in the bulk [22]. In this case, the Bekenstein-Hawking entropy $S_{BH} = N_{\rm sur}$ takes into account the gravitational holography properly. Then we obtain the relation

$$\rho_{\rm HM} = \frac{E^{\rm B}_{\rm del}}{(\delta l)^3} = \frac{m_{\rm p}^2}{l^2} \sim \rho_\Lambda \tag{13}$$

which shows that the holographic energy density could be derived from the covariant entropy bound and the spacetime fluctuations. Here we mention that Eq.(13) is consistent with the holographic model proposed in Ref.[18].

We check that $E^{\rm B}_{\rm del}$ may fit into a UV cell of size $l_{\rm p}$ in the holographic screen ($E^{\rm S}_{\rm del} \sim 1/l_{\rm p}$) as well. For this purpose, we consider the system which is composed of N UV cells [23]. Each cell has a Poissonian fluctuation in energy of amount $E_{\rm p} \sim 1/l_{\rm p}$. Then the root-mean-square fluctuation of energy will be

$$\Delta E_{\rm Po} = \sqrt{< (\Delta E_{\rm Po})^2 >} = \frac{\sqrt{N}}{l_{\rm p}} \tag{14}$$

[1] Actually, there are two approaches: bulk holography and holographic screens [22]. Two are closely related to each other as the UV-IR connection. In the bulk holographic approach, it is natural to postulate that uniformly distributed bulk holographic degrees of freedom are delocalized on the size l of the system. Then, the Heisenberg quantum energy of each delocalized holographic degrees of freedom is $E^{\rm B}_{\rm del} \sim 1/l$ with $\hbar = 1$. In this case, the quantum contribution to the global vacuum energy density is given by $\delta\Lambda_4 \sim E^{\rm B}_{\rm del}\frac{N_{\rm sur}}{l^3}$. The total number of degrees of freedom $N_{\rm sur} = \frac{l^2}{l_{\rm p}^2}$ is determined by the gravitational holography. Here we observe an important relation between bulk holographic and spacetime foam approaches: $\frac{N_{\rm sur}}{l^3} = \frac{1}{l_{\rm p}^2 l} = \frac{1}{(\delta l)^3}$. Consequently, one finds $\delta\Lambda_4 = \rho_\Lambda$.

which fits into the UV cell but it is proportional to the factor $\sqrt{N}$. This provides the energy density

$$\rho_{\rm Po}=\frac{\Delta E_{\rm Po}}{l^3}=\frac{\sqrt{N}}{l_{\rm p}l^3}. \tag{15}$$

Choosing $N=N_{\rm sur}$, we arrive at the holographic energy density

$$\rho_{\rm Po}=\frac{\sqrt{N_{\rm sur}}}{l_{\rm p}l^3}=\frac{1}{l_{\rm p}^2l^2}. \tag{16}$$

Hence, we observe that $\sqrt{N_{\rm sur}}E^{\rm B}_{\rm del}$ fits into a UV cell on the holographic screen as

$$\sqrt{N_{\rm sur}}E^{\rm B}_{\rm del}\simeq\frac{1}{l_{\rm p}}\sim E^{\rm S}_{\rm del}. \tag{17}$$

This indicates how IR fluctuations in the bulk can fit into UV cells on the screen. Also the conversion factor $\sqrt{N_{\rm sur}}$ could be easily explained by introducing a screen-bulk redshift factor of $1/\sqrt{g_{00}}$. The apparent horizon is a surface of infinite redshift, so a regulated screen must be employed. The Planck length and energy may be taken as UV cutoffs of local screen degrees of freedom. In this case, the inverse of screen-bulk redshift factor,

$$\sqrt{g_{00}}\sim\frac{l_{\rm p}}{l}\sim\frac{1}{\sqrt{N_{\rm sur}}} \tag{18}$$

gives a bulk quantum energy $E^{\rm B}_{\rm del}$. The definite connection is given by UV-IR connection as [24, 25]

$$E^{\rm B}_{\rm del}=\sqrt{g_{00}}E^{\rm S}_{\rm del} \tag{19}$$

which confirm Eq.(17) clearly.

On the other hand, from the energy bound of Eq.(11), it seems difficult to derive the holographic energy density because we do not know a form of $\Delta E_{\rm RWM}$ as

$$\rho_{\rm RWM}=\frac{\Delta E_{\rm RWM}}{(\delta l)^3}=\frac{\Delta E_{\rm RWM}}{(l_{\rm p}l)^{3/2}}. \tag{20}$$

Assuming that $\Delta E_{\rm RWM}\sim1/\sqrt{l_{\rm p}l}$ [2], one finds that $\rho_{\rm RWM}\sim\rho_\Lambda$. However, it is unclear why the energy of spacetime fluctuations is inversely proportional to the geometric mean of $\sqrt{l_{\rm p}l}$ of distance l and Planck length $l_{\rm p}$ when the energy bound is working for ordinary matter. In order to explain this, we introduce two length scales $l_{\rm UV}=l_{\rm p}$ and $l_{\rm IR}=l$

[2]According to Ref.[17], $\rho_{\rm RWM}$ is bounded between $(ll_{\rm p})^{-2}$ and $l^{-5/2}l_{\rm p}^{-3/2}$. Hence, this assumption is likely to be accepted.

by assuming that there is no connection between them. This means that we will not introduce any bound. Two energy densities of UV and IR scales are given by [23]

$$\rho_{\rm UV}=\frac{1}{l_{\rm p}^4}~~{\rm and}~~\rho_{\rm IR}=\frac{1}{l^4}. \tag{21}$$

Here $\rho_{\rm UV}$ determines the highest possible energy density in the universe, while $\rho_{\rm IR}$ determines the lowest possible energy density. Then the geometric mean (GM) of two energy densities takes the form

$$\rho_{\rm GM}=\sqrt{\rho_{\rm UV}\rho_{\rm IR}}=\frac{1}{l_{\rm p}^2 l^2} \tag{22}$$

which is just a form of holographic energy density ρ_Λ. Importantly, the geometric mean[3] of two length scales leads to the minimum length of the RWM: $l_{\rm GM}=\sqrt{l_{\rm UV}l_{\rm IR}}=\sqrt{l_{\rm p}l}\to\delta l$. Consequently, the geometric mean of two energies leads to

$$E_{\rm GM}=\sqrt{(l^3_{\rm UV}\rho_{\rm UV})(l^3_{\rm IR}\rho_{\rm IR})}=\frac{1}{\sqrt{l_{\rm p}l}}. \tag{23}$$

This is what we expect to obtain for the energy for the RWM. That is, if $\Delta E_{\rm RWM}=E_{\rm GM}$, one could obtain the holographic energy density from the RWM which is known to describe the ordinary matter. At this time, we do not prove that the presumed proposition of $\Delta E_{\rm RWM}=E_{\rm GM}$ is correct. Here we could support this by the dimensional argument.

We mention cumulative effects of spacetime fluctuations [17]. If successive fluctuations are completely anti-correlated (negative correlation: NC), the fluctuation distance δl is given by $l_{\rm UV}=l_{\rm p}$, being independent of the size of distance l. If successive fluctuations are completely correlated (positive correlation: PC), the fluctuation distance δl is given by $l_{\rm IR}=l$, the size of distance l. The zero correlation (ZC) corresponds to the RWM of $\delta l\sim\sqrt{l_{\rm p}l}$, while the order of correlation ($\delta l\sim(l_{\rm p}^2 l)^{1/3}$) for the HM is between NC and ZC. This implies that the effects of quantum gravity is strongest for UV (NC), while the effects of quantum gravity is zero for the RWM (ZC). We remind the reader that the quantum-gravitational effects of HM is between the strongest one and zero.

$$\frac{l_{\rm p}--(l_{\rm p}^2 l)^{1/3}---\sqrt{l_{\rm p}l}-----------l}{{\rm UV(NC)--HM--RWM(ZC,GM)------IR(PC)}}$$

The holographic uncertainty for the entropy bound leads to the holographic energy density. If this is unique, the entropy bound should be used for describing the system including self-gravitating effects only. Along this direction, we note that for the HM, the individual

[3]For comparison, we introduce two others: average (mean)$=\frac{l_{\rm p}+l}{2}\sim l$ and harmonic mean$=\frac{l_{\rm p}l}{2(l_{\rm p}+l)}\sim l_{\rm p}$ for $l_{\rm p}\ll l$. Hence, for $l_{\rm p}\ll l$, the relevant scale is the geometric mean.

fluctuations cannot be completely random, as opposed to the no correlation of RWM. Hence, successive fluctuations appeared to be entangled and somewhat anti-correlated as a result of effects of quantum gravity. On the other hand, the random-walk uncertainty for the energy bound could provide the holographic energy density by choosing $\Delta E_{\rm RWM} = E_{\rm GM}$. However, we do not know a close connection between RWM and GM

In addition, different sources may lead to the holographic energy density. These are vacuum fluctuation energy [23], entanglement entropy (energy), and Casimir energy [25, 26]. Until now, there is no unique way to give the holographic energy density.

3 Holographic and agegraphic dark energy models

Even though we got the holographic energy density, it is not guaranteed that the holographic energy density could describe the present accelerating universe. Here we choose

$$\rho_\Lambda = \frac{3c^2 m_{\rm p}^2}{L^2} \tag{24}$$

with a parameter c. In order for the holographic energy density to describe the accelerating universe, we have to choose an appropriate IR cutoff L. For this purpose, we may introduce three length scales of the universe: the apparent horizon=Hubble horizon for flat universe, particle horizon, and future event horizon. The equation of state is defined by

$$w_{\rm i} = -1 - \frac{a}{3\rho_{\rm i}}\frac{d\rho_{\rm i}}{da} \tag{25}$$

with the scale factor a. For the presence of interaction between two matters, one may introduce either the native EOS [27] or the effective EOS [28]. The density parameter is defined by

$$\Omega_{\rm i} = \frac{\rho_{\rm i}}{3m_{\rm p}^2 H^2} = \Big(\frac{c}{HL_{\rm i}}\Big)^2. \tag{26}$$

Its evolution is determined by

$$\frac{d\Omega_{\rm i}}{dx} = -3w_{\rm i}\Omega_{\rm i}(1-\Omega_{\rm i}) \tag{27}$$

for the presence of $\rho_{\rm i}$ and the cold dark matter (CDM) $\rho_{\rm m}$ with $x = \ln a$.

When the CDM is present, the Hubble horizon $L_{\rm HH} = 1/H$ does not describe the accelerating universe because its equation of state $w_{\rm HH} = 0$ is the same as the CDM dose. Using the first Friedmann equation with $\rho_{\rm HH} = 3c^2 m_{\rm p}^2 H^2$ leads to $(1-c^2)H^2 = \rho_{\rm m}/3m_{\rm p}^2$ with $\rho_{\rm m} = \rho_{\rm m0}/a^3$. This provides $\rho_{\rm HH} \propto 1/a^3$, which implies $w_{\rm m} = 0 = w_{\rm HH}$ [29]. Furthermore, the first Friedmann equation implies $\Omega_{\rm HH}+\Omega_{\rm m} = 1$ with $\Omega_{\rm HH} = c^2$. However,

this is an unwanted case because of $\Omega_{\rm m}=$ const. Using the second Friedmann equation (27), one has either $w_{\rm HH}=0$ or $\Omega_{\rm HH}=1$. On the other hand, one may find from Eq.(25)

$$w_{\rm HH}=-1-\frac{2\dot{H}}{3H^2} \tag{28}$$

which can be rewritten as

$$w_{\rm HH}=-1+\frac{2a\epsilon}{3} \tag{29}$$

with $\epsilon=-\frac{\dot{H}}{aH^2}$. For $H\simeq$ const, one finds $w_{\rm HH}=-1$. However, for $\epsilon>0$, $w_{\rm HH}>-1$, while for $\epsilon<0$, $w_{\rm HH}<-1$. This means that the holographic dark energy model with $L_{\rm HH}$ does not provide a promising EOS except the interacting case [30].

For the particle horizon with $L_{\rm PH}=a\int_0^a da'/a'^2H'$, it could not describe the accelerating phase because of

$$w_{\rm PH}=-1+\frac{2}{3L_{\rm PH}}\frac{dL_{\rm PH}}{dx}=-\frac{1}{3}+\frac{2\sqrt{\Omega_{\rm PH}}}{3c}\geq -1/3,\ \text{for } c\geq 1. \tag{30}$$

The only choice which provides an accelerating phase is the future event horizon $L_{\rm FH}=a\int_a^\infty da'/a'^2H'$ and thus its equation of state is given by

$$w_{\rm FH}=-1+\frac{2}{3L_{\rm FH}}\frac{dL_{\rm FH}}{dx}=-\frac{1}{3}-\frac{2\sqrt{\Omega_{\rm FH}}}{3c}\leq -1/3,\ \text{for } c\geq 1. \tag{31}$$

Hence, obtaining the accelerating phase is just the problem of choice of the IR cutoff L in the holographic dark energy models. This is because the logarithmic derivative of IR cutoff is given by

$$\frac{dL_{\rm PH/FH}}{dx}=L_{\rm PH/FH}\pm\frac{1}{H}. \tag{32}$$

That is, from Eqs.(25) and (32), the rate of change for the size L of universe determines the equation of state within the holographic dark energy model. If L is fixed, its EOS is -1 just like the cosmological constant. Hence L is rather ad hoc chosen. In other words, there is no such IR cutoff of future event horizon without first having a holographic dark energy and there is no holographic dark energy without first having an IR cutoff to define it. This leads to a conceptual paradox that is similar to the question of "the chicken and the egg" [31].

In order to understand this issue clearly, we introduce the agegraphic and new agegraphic dark energy densities [10, 32, 33, 34]

$$\rho_{\rm T}=\frac{3c^2m_{\rm p}^2}{T^2}\ \ \text{and}\ \ \rho_\eta=\frac{3c^2m_{\rm p}^2}{\eta^2} \tag{33}$$

with the same parameter c, respectively. At first sight, it seems to require the time scale of the universe. We remind the reader that we are working with the units of $c = \hbar = k_B = 1$. In this unit system, there is no essential difference between time and length. Thus one may use the terms like time and length interchangeably $(l = t)$, where $l_{\rm P} = t_{\rm P} = 1/m_{\rm P}$ being the reduced Planck length, time and mass, respectively. This means that $\rho_{\rm T}$ and ρ_η are the same as ρ_Λ except different IR cutoffs[4]. In this sense, we choose the IR cutoff as

$$T = \int_0^t dt' = \int_{-\infty}^x \frac{dx'}{H'} \tag{34}$$

which is the age of universe. In terms of length scale, it is the logarithmic integral of the Hubble radius H^{-1}. In addition, the conformal time is defined by

$$\eta = \int_0^t \frac{dt'}{a'} = \int_{-\infty}^x \frac{dx'}{a'H'} \tag{35}$$

which is the maximum comoving distance to a comoving observer's particle horizon since $t = 0$. That is, this is the logarithmic integral of the comoving Hubble radius $1/aH$. We call it the comoving horizon.

For the case that H is nearly constant, one finds that these are

$$T = \int_0^a \frac{da'}{a'H'} \simeq \frac{\ln[a]}{H} \equiv T_{\rm H},\ \ \eta = \int_0^a \frac{da'}{a'^2H'} \simeq -\frac{1}{aH} \equiv \eta_{\rm H}. \tag{36}$$

In this case, we have approximate forms of energy density

$$\tilde{\rho}_{\rm T} \simeq \frac{3c^2m_{\rm p}^2H^2}{(\ln[a])^2},\ \ \tilde{\rho}_\eta \simeq 3c^2m_{\rm p}^2a^2H^2,\ \ \tilde{\rho}_{\rm PH/FH} \simeq 3c^2m_{\rm p}^2H^2 = \rho_{\rm HH}, \tag{37}$$

which shows that the energy densities of proper distance $L_{\rm PH/FH}$ are approximately the same as that of the Hubble horizon, $\rho_{\rm HH}$. This implies that the holographic energy density model with the proper distance may be regarded as a "dynamical cosmological constant model".

The derivatives of these lead to the same expression, respectively

$$\frac{dT}{dx} = \frac{dT_{\rm H}}{dx} = \frac{1}{H},\ \ \frac{d\eta}{dx} = \frac{d\eta_{\rm H}}{dx} = \frac{1}{aH} \tag{38}$$

which shows that in calculating their EOS, there is no significant difference even for choosing $H \simeq$ const. Namely, the instantaneous rate of change for T and η are given by the Hubble radius and comoving Hubble radius, respectively. Using Eq.(25), we have

$$w_{\rm T} = -1 + \frac{2\sqrt{\Omega_{\rm T}}}{3c},\ \ w_\eta = -1 + \frac{2e^{-x}\sqrt{\Omega_\eta}}{3c}. \tag{39}$$

[4]For example, we have the present age of the universe $t_0 = \int_0^{t_0} dt'$, the Hubble horizon $H_0^{-1} = \frac{3}{2}t_0$, and the particle horizon $L^0_{\rm PH} = a_0 \int_0^{t_0} \frac{dt'}{a'} = 3t_0$[35]. The distance that the light travels is greater than we could get by naively multiplying the age of universe by the speed of light.

On the other hand, we may introduce

$$\bar{T} = \int_t^\infty dt' = \int_a^\infty \frac{da'}{a'H'} = \int_x^\infty \frac{dx'}{H'} \tag{40}$$

to be the future age of universe. Also

$$\bar{\eta} = \int_t^\infty \frac{dt'}{a'} = \int_a^\infty \frac{da'}{a'^2H'} = \int_x^\infty \frac{dx'}{a'H'}. \tag{41}$$

is the comoving distance to a comoving observer's future event horizon. Then their derivatives are given by opposite signs to T and η,

$$\frac{d\bar{T}}{dx} = -\frac{1}{H}, \quad \frac{d\bar{\eta}}{dx} = -\frac{1}{aH}. \tag{42}$$

Their equations of state are given by

$$w_{\bar{\mathrm{T}}} = -1 - \frac{2\sqrt{\Omega_{\mathrm{T}}}}{3c}, \; w_{\bar{\eta}} = -1 - \frac{2e^{-x}\sqrt{\Omega_\eta}}{3c}. \tag{43}$$

We note that $L_{\mathrm{PH}} = a\eta$ is the proper distance to particle horizon, whereas $L_{\mathrm{FH}} = a\bar{\eta}$ is the proper distance to future event horizon. L_{FH} is the distance to the most distance event we will ever see (the distance light can travel between now and the end of time) in contrast to L_{PH}, which is the distance to the most distant object we can currently see (the distance light has travelled since the beginning of time). An externally expanding model possesses future event horizon if light can not travel more than a finite distance in an infinite time, $\bar{\eta} < \infty$ [36]. However, we do not have the future event horizon for the future age of universe because of $\bar{T} \sim \infty$. Thus we exclude this case from our consideration.

For the choice of proper distance, we have non-accelerating phase for particle horizon, while we have the accelerating phase for future event horizon with $c \geq 1$. On the contrary to this, for the choice of coordinate distance (η=comoving distance), we have accelerating phase for particle horizon, while we have super-accelerating (phantom) phase for future event horizon with any c. This shows the apparent difference between holographic and new agegraphic dark energy models. However, there is no essential difference between two models. The apparent difference is to choose a different distance.

The causality issue may be resolved for agegraphic and new agegraphic dark energy model when choosing the coordinate distance. This is possible because the conformal time η as an IR cutoff exists in the new agegraphic dark energy model, irrespective of the existence of the eternal accelerated expansion in the future [32]. On the other hand, this issue arises for the holographic dark energy model with the proper distance. This is because in order to have an accelerating universe, one chooses the future event horizon

which shows the eternal accelerated expansion of the universe in the future. However, an accelerating phase may arise as a pure interaction phenomenon if pressureless dark matter is coupled to holographic dark energy whose IR cutoff scale is set by the Hubble length [37].

Finally, we would like to mention that the causality issue may be not resolved for the new agegraphic dark energy model in the future. The coordinate (comoving) distance induces more acceleration than the proper distance. Actually, we observe that $w_\eta \to -1$, irrespective of c in the future [33]. This implies the presence of the future event horizon because the accelerating phase of $-1/3 < w \leq -1$ could develop the future event horizon in the future [36].

4 Discussions

The spacetime foam model could provide the holographic energy density. However, its holographic model which implies the exotic matter, a dark energy with infinite statistics is not a unique way to derive the holographic energy density.

Furthermore, even if one gets the form of holographic energy density, it is a separate issue to find an accelerating universe from this density. Hence we may choose IR cutoff to be a dynamical length scale like either coordinate distance (age of universe T and comoving distance η) or proper distance (particle horizon $L_{\rm PH}$ and future event horizon $L_{\rm FH}$). The cases of comoving distance η and proper distance $L_{\rm FH}$ could explain an accelerating phase of the universe. However, it is unclear which distance is appropriate for the description of a dark-energy dominated universe. Along this direction, the proper distance of particle horizon $L_{\rm PH}$ was used to calculate the entropy bound [38, 39].

Until now, we do not know the nature of an exotic matter which may derive an accelerating universe because both of ordinary and exotic matters could lead to the holographic energy density as well as it is a matter of choice of IR cutoff to obtain an accelerating universe.

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